A Kernel Ridge Regression Model for Respiratory Motion Estimation in Radiotherapy

T. Geimer[1,2], A. Birlutiu[3], M. Unberath[1,2], O. Taubmann[1,2], C. Bert[2,4], A. Maier[1,2]

[1] Pattern Recognition Lab, Department of Computer Science, Friedrich-Alexander-Universität Erlangen-Nürnberg
[2] Graduate School in Advanced Optical Technologies (SAOT), Friedrich-Alexander-Universität Erlangen-Nürnberg
[3] Computer Science Department, "1 December 1918" University of Alba Iulia, Romania
[4] Department of Radiation Oncology, Universitätsklinikum Erlangen, Friedrich-Alexander-Universität Erlangen-Nürnberg

Abstract

In radiotherapy, breathing motion can be compensated by pre-trained motion models estimating the target motion from external surrogates [1][2]. We introduce Kernel Ridge Regresson to process high-dimensional surrogate data without the need for prior dimensionality reduction. The proposed model is compared to a related approach [3] with dimensionality reduction in the form of principal component analysis. Evaluation was performed in a simulation study based on nine 4D CT patient data sets achieving a mean estimation error of 0.84 ± 0.21 mm for our approach.

Introduction

Respiratory motion affects external beam radiation therapy

- Dose distribution from treatment plan based on CT image
- Motion: dislocation of target / survival of malignant cells

Motion estimation for gating / tracking

- Based on implanted fiducial and surface markers
- Only sparse information

High-dimensional data

- Dense information
- Computationally challenging without dimensionality reduction

Materials and Methods

Data Matrices

- Internal Motion: demons-based non-rigid registration [4] on 4D CT and cropped to internal region of interest (**Fig. 1a**)
  $\{\boldsymbol{t}_1, \dots, \boldsymbol{t}_n\} \in \mathbb{R}^{d_t}$ stored column-wise in $\boldsymbol{T} \in \mathbb{R}^{d_t \times n}$
- RI: motion fields interpolated at reference surface (**Fig. 1b**)
- FL: Digitally Reconstructed Radiographs using CONRAD [5]
  $\{\boldsymbol{s}_1, \dots, \boldsymbol{s}_n\} \in \mathbb{R}^{d_s}$ stored column-wise in $\boldsymbol{S} \in \mathbb{R}^{d_s \times n}$

Kernel Ridge Regression (KRR)

- Objective function: $\arg\min_{\boldsymbol{W}} \left(\frac{1}{2}\|\boldsymbol{WS} - \boldsymbol{T}\|_F^2 + \alpha \frac{1}{2}\|\boldsymbol{W}\|_F^2\right)$
- Prediction: $\boldsymbol{t}_{pred} = \boldsymbol{T}\,(\boldsymbol{K} + \alpha\,\boldsymbol{I}_n)^{-1}\boldsymbol{\kappa}(\boldsymbol{s}_{new})$
  - $\boldsymbol{K}_{ij} = \boldsymbol{\phi}(\boldsymbol{s}_i)^\top \boldsymbol{\phi}(\boldsymbol{s}_j)$ Gram matrix of mapped samples
  - $\boldsymbol{\kappa}(\boldsymbol{s}_{new})_i = \boldsymbol{\phi}(\boldsymbol{s}_i)^\top \boldsymbol{\phi}(\boldsymbol{s}_{new})$ Kernel response for new surrogate
- Implicit mapping $\boldsymbol{\phi}$ expressed only in terms of inner products
- Supports non-linear mappings, e.g. Gaussian kernel

Comparison to Principal Component Regression (PCR) [3]

- Principal Component Analysis to decompose a given data set into mutually orthogonal modes of variation

$$\boldsymbol{F}_T \in \mathbb{R}^{p_t \times n}, p_t \ll d_t$$
$$\boldsymbol{F}_S \in \mathbb{R}^{p_s \times n}, p_s \ll d_s$$

- Multi-linear regression on feature weights

Evaluation

- 9 time-resolved 4-D CT patient data sets ($0.97 \times 0.97 \times 2.5\ mm^3$)
- Leave-one-phase-out cross-evaluation: mean estimation error

Contact

tobias.geimer@fau.de

http://www5.cs.fau.de/~geimer

The authors gratefully acknowledge funding of the Erlangen Graduate School in Advanced Optical Technologies (SAOT) by the German Research Foundation (DFG) in the framework of the German excellence initiative.

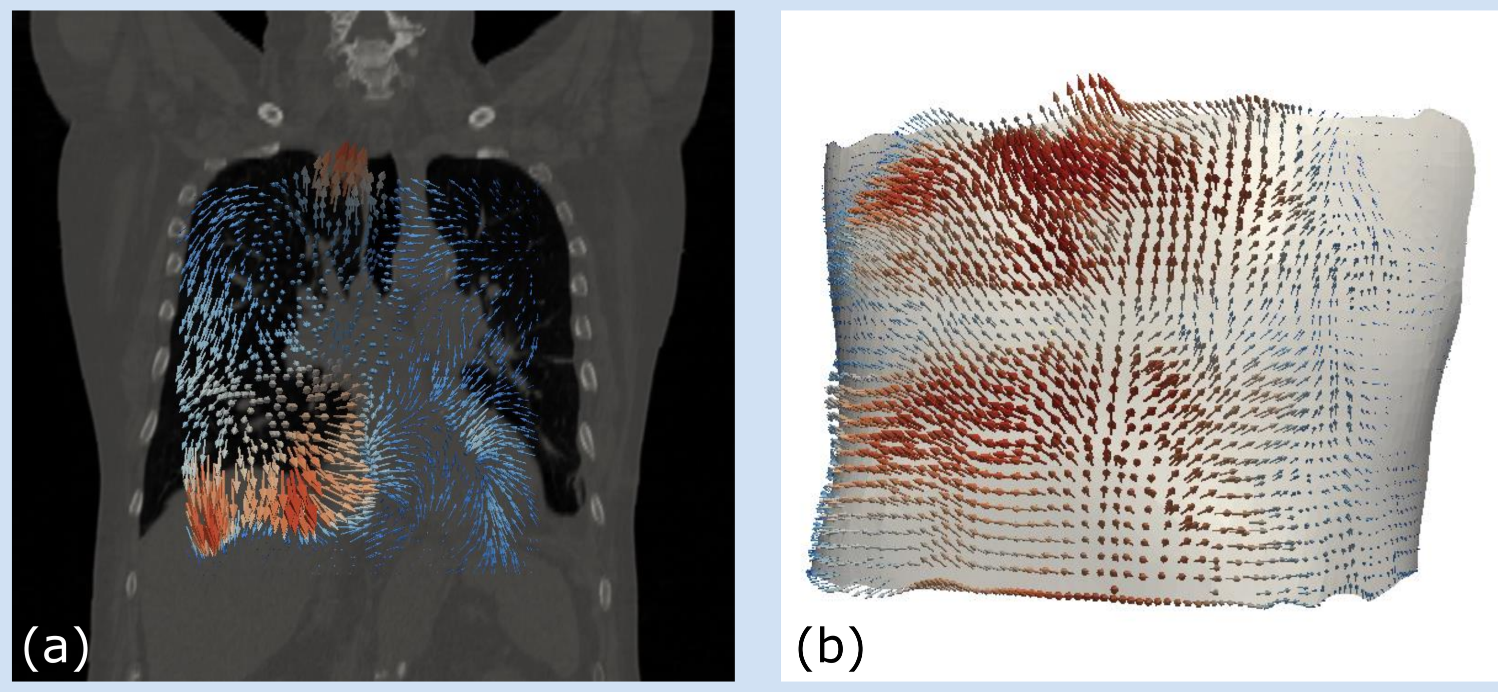

Figure 1: Qualitative representation of 3-D motion fields obtained by non-rigid registration: **(a)** cropped to the internal ROI and **(b)** interpolated at the extracted surface mesh.

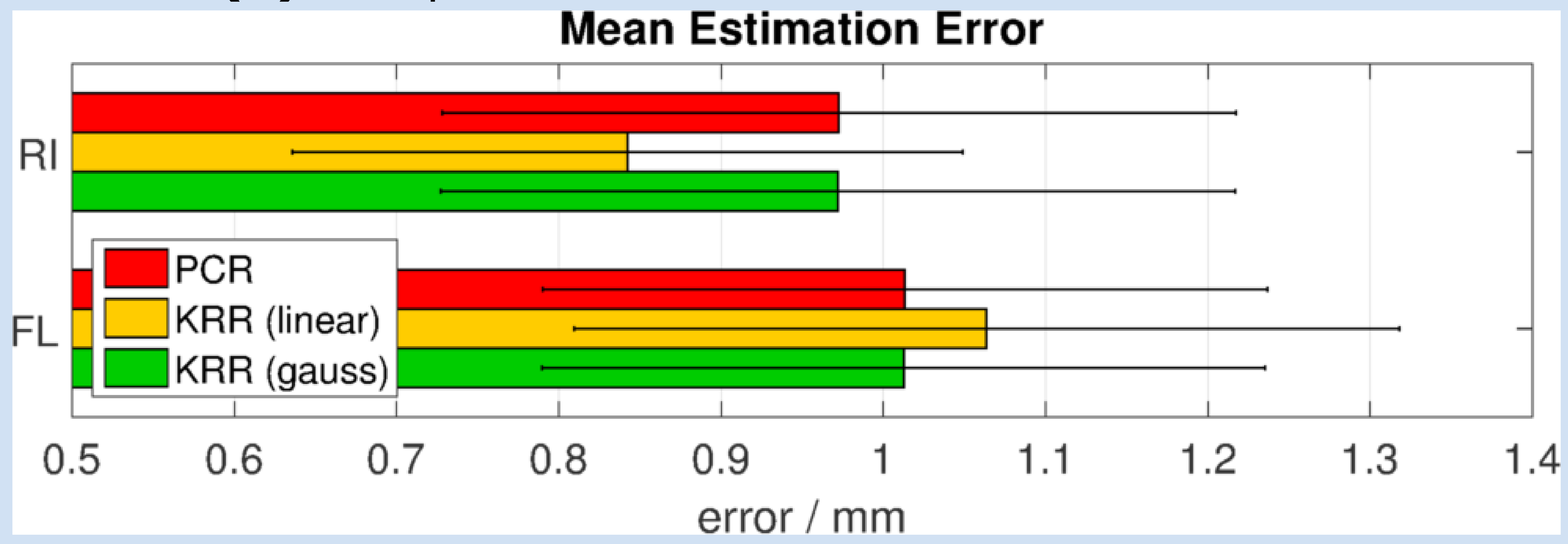

Figure 2: Mean error and standard deviation over all patients based on surface (RI) and fluoroscopic (FL) surrogate using Principle Component (PCR) and Kernel Ridge Regression (KRR).

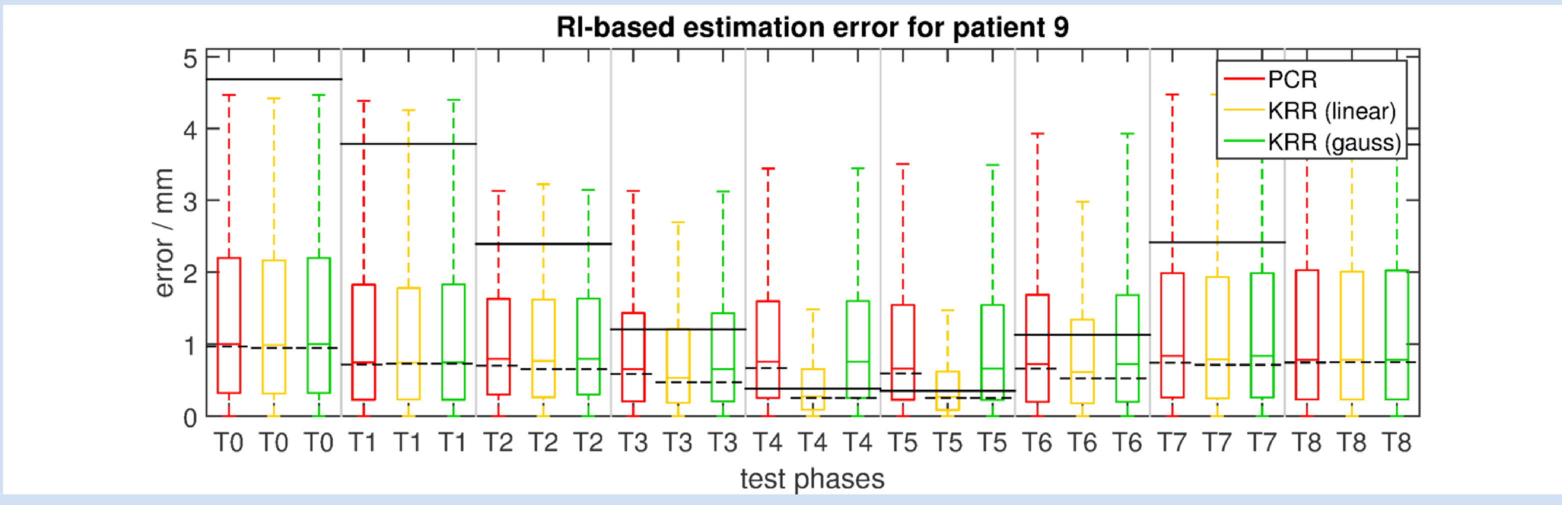

Figure 3: Surface-based estimation error per phase for patient 9. Black bars: mean magnitude of ground truth deformation field. Dashed lines: lower model bound.

Results and Discussion

Results

- Reference mean magnitude: 2.48 ± 0.81 mm
- All proposed methods suitable for compensation at around 1.0 ± 0.22 mm estimation error (**Fig. 2**)
- Best: 0.81 ± 0.21 mm for KRR with a linear kernel

Discussion

- No improvement from non-linear KRR over PCR
- Phase reconstruction
  - KRR: weighted sum of observed training samples
  - PCR: linear combination of eigenvectors
- Only surface-based linear KRR capable of explaining phases near end-exhale (**Fig. 3**)

Conclusions

- Motion estimation operating directly on observed surrogate data without prior dimensionality reduction

Future Work

- Further evaluation closer to the application case
- Training on Planning CT / Testing on Follow-up CT

References

[1] McClelland, J. R. et al., Med Image Anal, 17(1):19-42, (2013)
[2] Wilms, M. et al., Phys Med Biol, 64(5), (2014)
[3] Taubmann, O. et al., Proc. 28th CARS, p.33-34, (2014)
[4] Fischer & Modersitzki, Linear Algebra Appl., 380(0):107-124, (2004)
[5] Maier, A. et al., Med Phys, 40(11):111914-1-8, (2013)